

# Technical Notes

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## Theoretical Study of Supersonic Flow Separation over a Rearward-Facing Step

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### Introduction

THE study of supersonic flow separation over a rearward-facing step has received considerable attention because of its potential application to the supersonic combustion ramjet (scramjet) engine development. In previous experimental works,<sup>1–8</sup> various flow properties including base-pressure characteristics and turbulence intensities were measured to study the flow structure of supersonic reattaching shear layers. Theoretically, several numerical models<sup>9–15</sup> were employed to solve the supersonic reattachment flowfield with considering the effects of heat transfer and chemical reaction. In this analysis, the lower-upper symmetric successive overrelaxation (LU–SSOR) method<sup>16–18</sup> with a finite volume approach has been adopted. As a first step toward a thorough simulation of the combustion phenomena associated with a supersonic sudden-expansion flow, the purpose of the present Note is to investigate the effects of incoming boundary-layer thickness, inlet Mach number, and step height on the cold flow structures behind a rearward-facing step.

### Procedure of Calculation

The physical model considered a two-dimensional supersonic airflow over a rearward-facing step. The governing equations, consisting of conservation of mass, momentum, and energy, were solved numerically employing a flux-vector splitting LU–SSOR scheme. The algebraic eddy-viscosity model of Baldwin–Lomax was adopted for turbulence closure.<sup>19</sup> Detailed descriptions of the theoretical formulation and the numerical method are given in Ref. 20.

In this study, uniform grid spacings were used in the horizontal direction. However, a vertically clustered grid setup was adopted near the separation corner and bottom wall to resolve steep variations of flow properties. For most calculations, an  $80 \times 60$  grid system was employed, and the physical dimension of the computational domain was 215.9 mm in length and 76.2 mm in height. To conduct grid independency study, one calculation using a  $94 \times 70$  grid was performed. The solutions showed negligible improvement over the  $80 \times 60$  grid. This observation may conclude that the computational results are relatively independent of the grid density. For each calculation, the normalized residual errors of dependent variables converged to  $10^{-8}$ .

### Discussion of Results

Figure 1 presents vector plots of the calculated mean flow pattern, Mach number, and pressure contours for the baseline case, a Mach 1.83 airflow with a uniform axial velocity profile and a step height of 25.4 mm. Based on the calculated results, it is quite obvious that the overall flowfield can be characterized by several different regions: corner expansion region, free shear layer, recirculation zone, and recompression region. Emerging from downstream near the reattachment point, a set of closely spaced contours in Mach number plot clearly indicates the location of a wake shock. The shear layer separates the recirculation zone from the main flow. A large pressure drop occurs due to the expansion process around the corner. The base pressure is found to be relatively constant near the step. Since the damping effect introduced by the explicit artificial dissipation could smooth the final solution, the lip shock was not observed in calculations.

To perform the code validation, the present computer program was also adopted to calculate the flow structure of a Mach 1.83 flow over a rearward-facing step with a height of 25.4 mm, which was experimentally investigated in Ref. 7. The turbulent boundary-layer thickness of the incoming stream was 8 mm. The stagnation pressure and temperature were 382 kPa and 291 K, respectively. A comparison of the predicted and measured surface pressure distributions along the bottom wall is given in Fig. 2. The surface pressure was normalized to the inlet freestream pressure. Both experimental data and predicted surface-pressure distribution show the same trend in that the wall pressure is relatively constant within approximately 1.5 step height before it begins its recovering process. However, some discrepancy exists at the base region. This is believed to be caused by the use of the algebraic eddy viscosity model.

To investigate the influence of various geometrical and fluid dynamic parameters on the flowfield characteristics in a rearward-facing step, four cases (based on different step heights, inlet freestream Mach numbers, and incoming boundary-layer thickness) have been computed at the same freestream pressure of 63,500 Pa and freestream temperature of 174.28 K. These four cases are summarized in Table 1. In case 2, the inlet boundary-layer thickness is 8 mm, and the Reynolds number based on this thickness is  $6.44 \times 10^5$ . The one-seventh-law was adopted in case 2 to specify the velocity profile inside the incoming boundary layer, whereas the uniform velocity distribution was employed for the freestream region.

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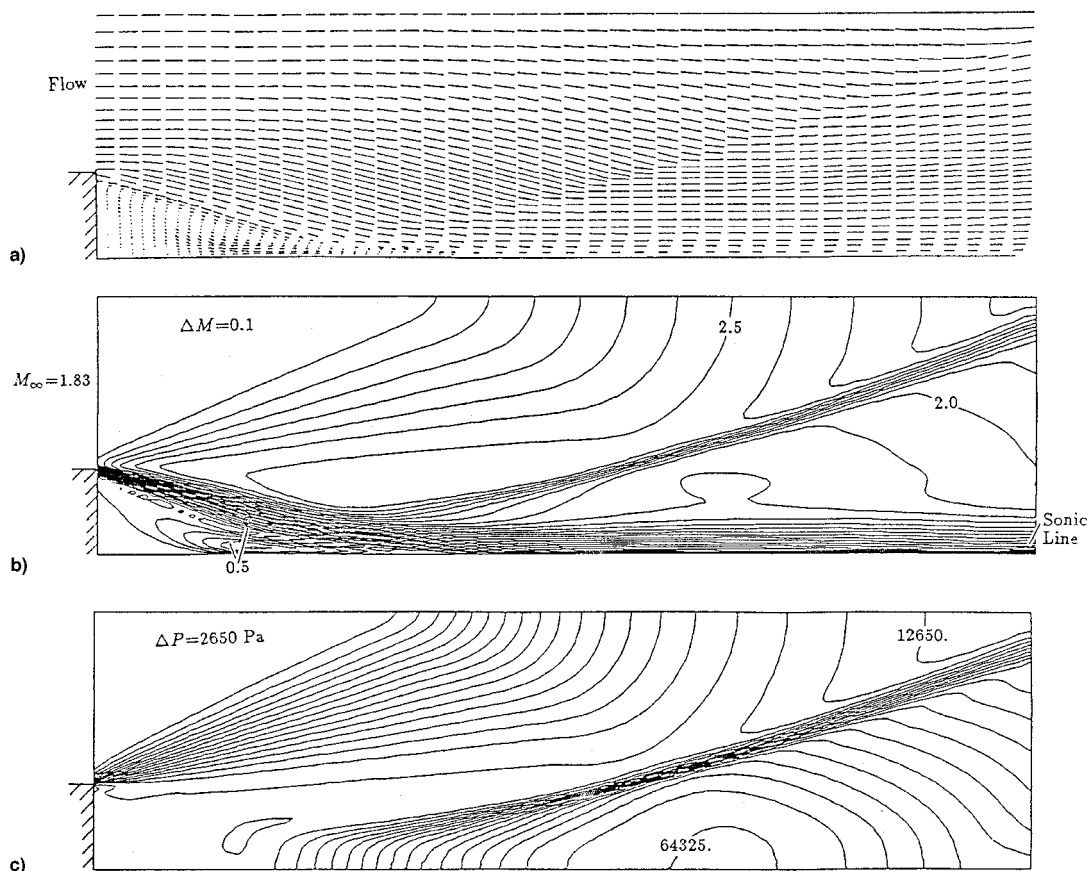


Fig. 1 a) Vector plot, b) Mach number, and c) pressure contour for inlet Mach 1.83 at a step height of 25.4 mm.

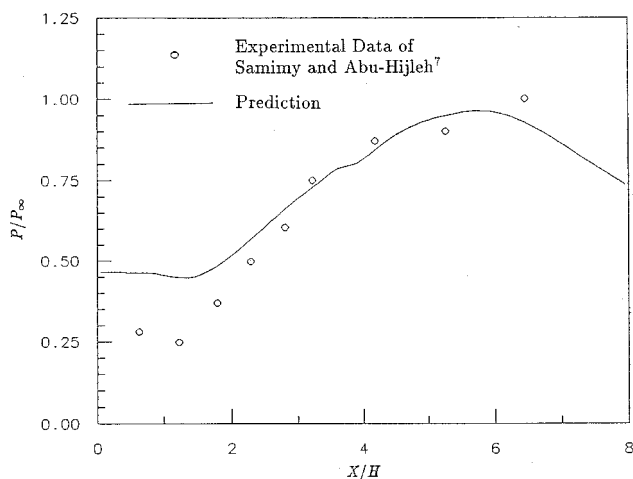


Fig. 2 Calculated and measured static pressure distribution along the bottom wall.

Since the size and flow properties of the recirculation zone could strongly affect the ignition and combustion processes of the overall combustor, the volume of this recirculating region, which typically can be determined from the reattachment point, is of particular interest to combustion engineers. The predicted reattachment points for all four cases in Table 1 were found to be located at 2.68, 3.39, 2.94, and 1.95 step heights, respectively. These calculated results indicate that 1) the reattachment length of the flow with the boundary-layer effect is larger than that of the baseline case, 2) the reattachment point moves upstream as the inlet freestream Mach number increases, and 3) the size of the recirculation zone increases with increasing the step height. In terms of the effect of the free-stream Mach number on the size of the recirculating region,

Table 1 Flow conditions used in the parametric study

Case no.	$H$ , mm	$M_\infty$	With or without inlet boundary-layer effect
1	25.4	1.83	Without
2	25.4	1.83	With
3	12.7	1.83	Without
4	12.7	3.50	Without

the physical explanation is as follows. The higher the free-stream Mach number, the stronger the corner expansion fan, thus, the lower the base pressure. This causes a stronger suction that bends the main flow downward. Therefore, the size of the recirculation zone decreases as the freestream Mach number increases. It is useful to note that the previous trend is true only for supersonic freestream conditions. For subsonic laminar flow over a rearward-facing step, the reattachment length increases rapidly with the increase of flow inertia, according to the review paper by Eaton and Johnston.<sup>21</sup>

In the practical design of scramjet propulsion systems, base pressure could be another important parameter for the consideration of ignition and flameholding processes. Figure 3 displays the effects of incoming boundary layer, step height, and inlet freestream Mach number on the static pressure distribution along the bottom wall. As shown in Fig. 3a, the boundary-layer effect at the corner causes a lower local Mach number near the corner regions; hence, the weaker the expansion fan and the higher the base pressure. This predicted trend is in accordance with Power's experimental observation.<sup>4</sup> In the dimensionless axial coordinate, the static pressure distributions along the bottom wall are quite similar for two different step heights (see Fig. 3b). However, the physical pressure distributions are quite different. The change in inlet Mach number

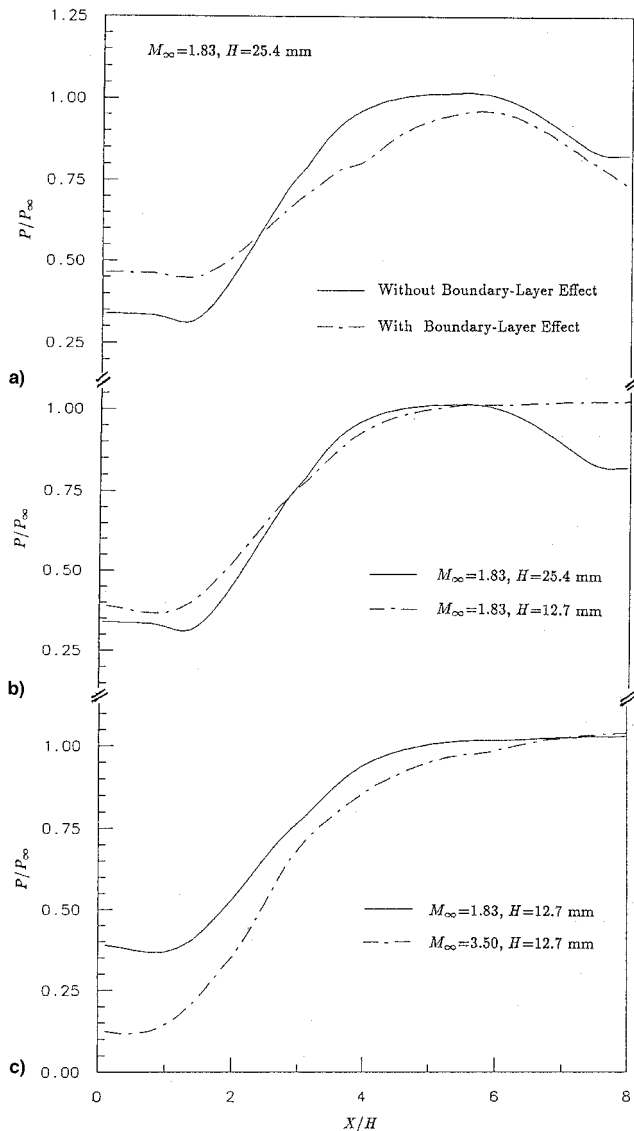


Fig. 3 Effects of a) incoming boundary layer, b) step height, and c) inlet Mach number on the static pressure distribution along the bottom wall.

from 1.83 to 3.5 decreases the base pressure significantly, as shown in Fig. 3c. From the design methodology point of view, the gross engine thrust is directly proportional to total pressure. Therefore, the incoming flow to the supersonic combustor of a hypersonic engine requires a higher combustor inlet Mach number to avoid total pressure loss, and thereby to achieve a higher thrust. Unfortunately, this requirement often causes technical difficulties in achieving successful ignition and/or in attaining combustion stability due to the very low base pressure and small-sized recirculation zone serving as the flameholder. In view of these complicated interactions between the operating conditions of inlet and combustor, the design of these components must be compromised to accomplish optimum performance.

### Summary and Conclusions

A model based on a set of two-dimensional Navier-Stokes equations has been solved numerically using a flux-vector splitting LU-SSOR scheme to study the flow structure of supersonic flow over a rearward-facing step. Reasonable agreement between calculated results and the reported data of surface static pressure distribution was obtained. The effects of the incoming boundary layer, step height, and inlet freestream

Mach number on the flow characteristics were also studied in detail. Simulation results demonstrate that both the recirculation zone and base pressure tend to decrease when the free-stream Mach number is increased or the incoming boundary-layer thickness is reduced. Reduction of the base pressure and recirculation zone could cause difficulties in ignition of flame-holding processes.

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